NP-Hardness of the Sorting Buffer Problem on the Uniform Metric*

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Abstract. An instance of the sorting buffer problem consists of a sequence of requests for service each of which is specified by a point in a metric space (V, d), where V is a set of points and d is a distance function, a server which moves from point to point in order to serve these requests, and a finite-capacity sorting buffer capable of storing up to k requests. To serve a request, the server needs to visit the request point where serving a request $p \in V$ following the service to a request $q \in V$ requires the cost corresponding to the distance between p and q, i.e., $d(p, q)$. The sorting buffer which is a random access buffer can be used to reorder the input sequence. Whenever a new request arrives, it has to be stored in the sorting buffer. At any point in time, the sorting buffer contains the first k requests of the input sequence that have not been served so far, and one of those k requests can be served by moving the server to it and removing it from the sorting buffer. The objective of the sorting buffer problem is to serve all input requests in a way that minimizes the total distance traveled by the server in the metric space by reordering the input sequence.

The *buffering-reordering mechanism* is widely used in many applications and very universal. Thus, several metric spaces are investigated in the literature: Räcke, Sohler, and Westermann [RSW02] first introduced the sorting buffer problem on a *uniform metric*, in which points are either at distance 0 or 1. They modeled the following paint shop scenario: An input is a sequence of cars with specific colors. If consecutive two cars must be painted in different colors, then a significant set-up and cleaning costs are incurred in changing colors. That is, the consecutive two requests are at distance 1 if the corresponding cars have to be painted in different colors, and at distance 0 otherwise. Kohrt and Pruhs [KP04], and Bar-Yehuda and Laserson [BL05] also consider the problem on the uniform metric and provide constant-factor approximation algorithms, but their objective is to maximize the reduction in the cost from that of the input sequence. Khandekar and Pandit [KP06] investigate a *line metric*, which is motivated by its application to a disc scheduling problem. Englert, Räcke, and Westermann [ERW07] study more general metric spaces.

For the general setting, the sorting buffer problem is known to be NP-hard by using a simple reduction from the minimum weight Hamiltonian path problem, and can be solved optimally using dynamic programming in $O(n^{k+1})$ time and in $O(n^{|V|+1})$ where n is the length of the input sequence. Little surprisingly, however, even for the uniform metric, it is not known if the problem remains NP-hard. In the talk, we present the first NP-hardness proof for the sorting buffer problem on the uniform metric. Moreover, we discuss the upper and the lower bounds of several simple strategies such as First In First Out (FIFO) and Last In First Out (LIFO).

References

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